

Two-loop snail diagrams: relating neutrino masses to dark matter

Yasaman Farzan¹

*Physics school, Institute for Research in Fundamental Sciences (IPM),
P.O. Box 19395-5531, Tehran, Iran.*

Abstract

Various mechanisms have been developed to explain the origin of Majorana neutrino masses. One of them is radiative mass generation. Two-loop mass generation is of particular interest because the masses and couplings of new particles propagating in the loop can be in the range testable by other experiments and observations. In order for the radiative mass suppression to be reliable, it should be guaranteed that lower loop contributions are suppressed. Based on loop topology and the form of electroweak presentation of the particles propagating in the loop, one can determine whether a lower—and therefore dominant—loop contribution is possible. We present a model based on these general considerations which leads to neutrino masses via a two-loop diagram which we dub as “snail-diagram”. The model has two natural candidates for dark matter one of them being a neutral Dirac fermion which can satisfy the conditions of the thermal freeze-out scenario by annihilation to lepton pairs. We comment on the possibility of explaining the GeV gamma ray excess observed by Fermi-LAT from the region close to the Galaxy Center. We also discuss possible signals at the LHC and at experiments searching for lepton flavor violating rare decays.

¹e-mail address: yasaman@theory.ipm.ac.ir

1 Introduction

Origin of neutrino masses and nature of Dark Matter (DM) are among the most compelling open questions in particle physics. In recent years, models in which neutrinos acquire their masses at loop level have received considerable attention (see Ref. [1–4] for a model-independent analysis). Within these models, the smallness of neutrino masses can be understood (at least partially) by loop suppression. If the new particles propagating in the loop are lighter than a few TeV, the resulting scheme will be phenomenologically interesting because in that case the new states can potentially be produced at the LHC. If this turns out to be the case, the radiative neutrino mass model can be tested at man-made accelerators. This is a great advantages over the “canonical” tree-level type-I seesaw model [5], for which on-shell production of the new states is inconceivable in any foreseeable future in man-made accelerators.

Assuming that the only source of electroweak symmetry breaking is the vacuum expectation of the Higgs, n -loop contributions to neutrino masses can be estimated as

$$m_\nu \sim \left(\frac{g^2}{16\pi^2} \right)^n \left(\frac{\langle H \rangle^2}{m_{\text{New}}} \right) \left[1, \left(\log \frac{\Lambda}{m_{\text{New}}} \right)^n \right] \quad , \quad (1)$$

where m_{New} is the mass scale characterizing the new physical degrees of freedom appearing in the loop and Λ is the ultraviolet (UV) cut-off scale of the model satisfying $\Lambda \gg m_{\text{New}}$. Taking $m_{\text{New}} \sim 1$ TeV, $m_\nu \sim 0.1 - 1$ eV [6–8], $\Lambda/m_{\text{New}} \sim 10$ and $n = 2$, we find that $g \sim 10^{-3}$. Increasing n , the required values of the couplings will of course increase. The same couplings also lead to Lepton Flavor Violating (LFV) processes. For $m_{\text{NEW}} < 10$ TeV, null results of searches for LFV rare decays of the muon and the tau lepton yield strong bounds on the combinations of such couplings. For $n = 2$, these bounds are naturally satisfied but for $n > 2$, a special mechanism such as the flavor symmetries suggested in [2] have to be invoked to make neutrino masses consistent with LFV bounds. From this perspective, the two-loop neutrino mass models seem more natural and are favored over higher order loop models.

In order to explain the smallness of neutrino masses through radiative schemes, one should make sure that lower—and therefore dominant—loop contributions are absent. In [2] based on general considerations of topological structure of the loops and symmetries, the requirements assuring the absence of lower order contributions have been systematically formulated. Here in this paper, using the “recipes and ingredients” outlined in [2], we reconstruct a model where neutrino masses are generated at the two-loop level through what we call “snail diagrams”.

Our model respects a new $Z_2 \times U(1)_{\text{New}}$ symmetry. These symmetries stabilize two of the lightest particles with non-trivial transformation under these discrete symmetries against decay. If these stable particles are neutral, they may be considered as a candidate for DM. In our model, a Dirac fermion, ψ which is a singlet under the electroweak symmetry plays the role of the DM. The DM couples to left-handed leptons via a Yukawa coupling. The abundance of ψ is determined by thermal freeze-out scenario via annihilation to lepton pairs. To avoid the severe bounds from LFV, we assume that ψ couples exclusively to only one flavor. An excess in the GeV range γ -ray has been reported in Fermi-LAT data on signal from regions close to galactic center. One of the solutions is dark matter of mass 10 GeV annihilating into tau pair [9]. Another possibility is annihilation into $b\bar{b}$ pair [10]. The dark matter origin of this signal has been however questioned and alternative sources have been suggested [11]. We will comment on the possibility of accommodating this scenario within our model.

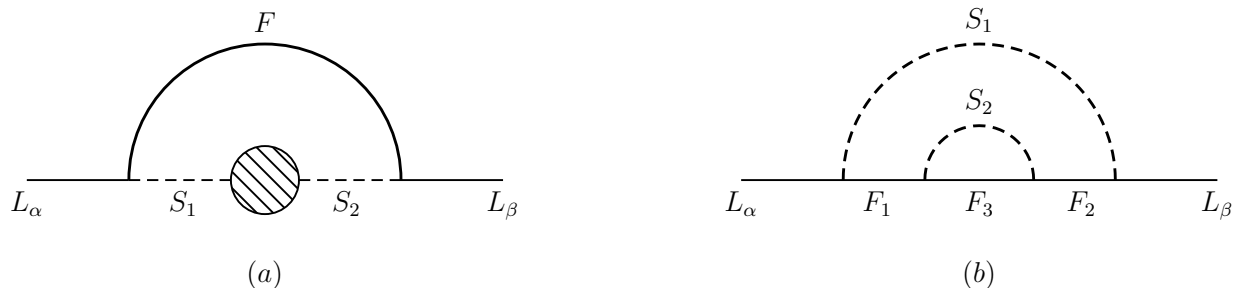


Figure 1: *Two-loop diagrams with one-loop wave function renormalization of scalar (a) and fermion (b) fields.*

The paper is organized as follows. In section 2, we generally discuss two-loop contributions to neutrino masses based on the topology of the diagrams. In section 3, we introduce the content of the model. In section 4, we discuss lepton flavor violating effects. In section 5, we calculate the contribution to neutrino masses. In section 6, we discuss the annihilation of dark matter pair and possibility of accommodating the claimed gamma ray excess from the region close to the galactic center. In sections 7 and 8, we respectively discuss signatures at the LHC and contribution to anomalous magnetic dipole moment. Conclusions are summarized in section 9.

2 Comments on two-loop neutrino masses: crab and snail diagrams

Two-loop diagrams contributing to neutrino masses have been systematically discussed in [2, 4]. Based on the topologies of the two-loop diagrams, they can be classified in two groups: (1) Diagrams with a one-loop sub-diagram that can be considered as a correction to one of the internal lines. Figs. (1- a) and (1- b) show corrections to internal scalar and fermion lines, respectively. The “bubble” on the scalar line may indicate a fermion loop, a scalar loop with trilinear scalar vertices or a scalar loop with quartic scalar vertex. Further details can be found in [2]. (2) Diagrams in which an internal line interconnects the scalar and fermion lines coming from the vertex connected to the external lines. These types of diagrams are rather well-known and have been employed in the literature to radiatively produce neutrino mass at the two-loop level. A pioneer work using such diagram is the famous Cheng-Li-Babu-Zee model [12–14].

In Ref. [2], it is argued that diagrams of type (1-a) contributing to the effective Weinberg operator

$$\mathcal{O}_5 \sim (L^T C i\tau_2 H) (H^T i\tau_2 L) , \quad (2)$$

can always be accompanied by a one-loop contribution to neutrino mass. The reason is that if the symmetries of the Lagrangian allow the one-loop internal sub-diagram, they will also allow a renormalizable term with which the internal loop can be replaced. Depending on where the two external Higgs lines are attached (vacuum insertions $\langle H \rangle$), these renormalizable terms can be $S_1 S_2$, $S_1 S_2 H$ or $S_1 S_2 H^2$.

On the contrary, the so-called rainbow diagrams generically depicted in Fig. (1-b) are not necessarily accompanied by any one-loop counterpart. The argument is based on the following

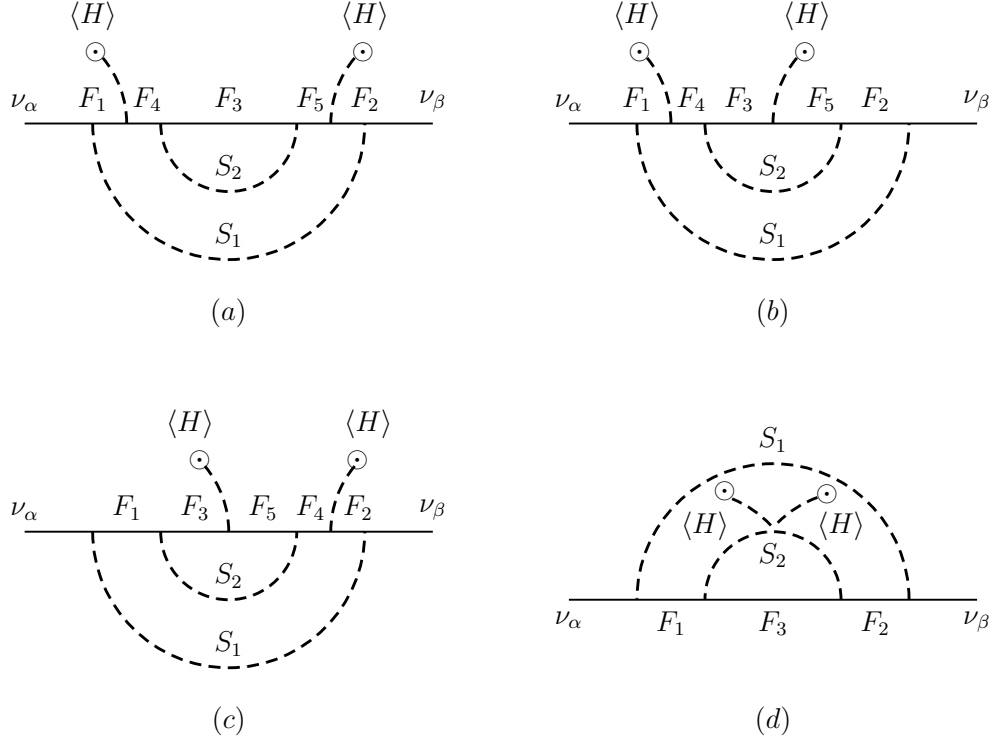


Figure 2: *Generic crab and snail diagrams. We have not specified on which fermionic line the chirality flip takes place.*

fact. While a term such as $S_1 S_2 H^2$ is renormalizable, its fermionic counterpart, $F_1 F_2 H^2$, is not. Thus, depending on the electroweak structure of the fermion lines attached to the internal loop (F_1 and F_2 in Fig. 1-(b)) and the way in which the Higgs external lines are attached to the corresponding diagram, there might or might not be a one-loop contribution.

For the sake of the following discussion, let us consider the diagrams in Fig. 2: “crab” (diagrams (a) – (c)) and “snail” diagrams (diagram (d)). The internal loops in “crab” diagrams can be respectively replaced by renormalizable vertices $F_4 F_5$, $F_4 F_2 H$ and $F_1 F_4 H$. “Crab” diagrams are therefore always accompanied by a leading one-loop contribution, and are in that sense irrelevant. For “snail” diagrams, instead, there is no such possibility because $F_1 F_2 H^2$, being non-renormalizable cannot appear in the Lagrangian. This argument of course holds under the assumption that neutrino masses are generated below the electroweak symmetry breaking scale only from Weinberg operator in Eq. (2). If we included a hypercharge -2 electroweak scalar triplet (Δ), with scalar interactions enabling a non-vanishing vacuum expectation value, $\langle \Delta \rangle \neq 0$, the external Higgs lines (vacuum insertions $\langle H \rangle$) could be replaced by a single triplet vacuum insertion $\langle \Delta \rangle$. In that case the internal loop could be replaced by the renormalizable vertex $\Delta F_1 F_2$.

In what follows we build a model where the effective Weinberg operator arises via a “snail” diagram.

3 Snail models

In this section, we present a model that can provide a suitable Dirac fermion DM and give mass to neutrinos via a two loop diagram. We first introduce the symmetry structure and field content of the model and then discuss why each assumption is made. In the next sections, we shall discuss the contribution to neutrino mass, annihilation of DM pairs to lepton pairs, effects on LFV and magnetic dipole moment of the muon and signals at the LHC.

The model is based on an unbroken $Z_2 \times U(1)_{NEW}$ symmetry. The SM particles are all even and neutral under this symmetry. The model also enjoys an approximate lepton number symmetry, $U(1)_L$ softly broken by a fermion mass mixing term. The field content of the model is shown in table 1.

	$SU(2)$	$U(1)_Y$	$U(1)_L$	$U(1)_{NEW}$	Z_2
F_1	d	-1	1	1	+
F_2	d	-1	1	-1	+
F_3	d	1	1	1	+
ψ	s	0	1	1	-
S	s	0	0	-1	+
Φ	d	-1	0	0	-
Φ'	d	-1	0	-1	-

Table 1: Field content of the model. By “d” and “s” in the second column we mean doublet and singlet, respectively. We have used the convention for hypercharge in which $Q = T^3 + Y/2$. The first four fields (*i.e.*, F_1 , F_2 , F_3 and ψ) are Dirac fermions and the last three lines (S , Φ and Φ') are scalar fields.

The new fermions are all Dirac particles and their masses are of form

$$\sum_i m_{F_i} \bar{F}_i F_i + m_\psi \bar{\psi} \psi .$$

As a result, neutral and charged components of F_i are degenerate. We also include mass term of form

$$m_M (F_{2R}^a)^T c F_{3R}^b \epsilon_{ab} + m'_M (F_{2L}^a)^T c F_{3L}^b \epsilon_{ab} + \text{H.c.} \quad (3)$$

which is supposed to be the only source of lepton number violation. The Yukawa couplings of the new particles symmetric under $Z_2 \times U(1)_{NEW} \times U(1)_L$ are

$$\begin{aligned} \mathcal{L}_{Yukawa} = & g_\alpha S^\dagger F_{1R}^\dagger L_\alpha + h_\alpha S F_{2R}^\dagger L_\alpha + Y_{R\alpha} \Phi'^\dagger \psi_R^\dagger L_\alpha + \\ & Y_1 \Phi^\dagger \psi_L^\dagger F_{1R} + Y_2 \epsilon_{ab} \Phi^a \psi_L^\dagger F_{3R}^b + Y'_1 \Phi^\dagger \psi_L^\dagger F_{1L} + Y'_2 \epsilon_{ab} \Phi^a \psi_L^\dagger F_{3L}^b + \text{H.c.} \end{aligned} \quad (4)$$

The new scalars can have interactions between themselves and SM Higgs. We assume that only the SM Higgs obtains a VEV so $U(1)_{NEW}$ and the new Z_2 symmetries remain unbroken. The Z_2 and $U(1)_{NEW}$ forbid mass terms mixing the scalars such as $H^\dagger \Phi$ or $\Phi^\dagger \Phi'$. We can however have couplings of form

$$(\lambda (H^a \Phi^b \epsilon_{ab})^2 + \text{H.c.}) \quad \text{and} \quad \lambda' |H^\dagger \Phi|^2.$$

The λ coupling after electroweak symmetry breaking will lead to a mass term of form $(\Phi^0)^2$ for the neutral component of $\Phi^0 \equiv (\phi_R + i\phi_I)/\sqrt{2}$. Thus, there will be a splitting between ϕ_R and ϕ_I . We however take λ to be real so these fields remain mass eigenstates. We will denote the masses of these components with m_I and m_R :

$$m_R^2 - m_I^2 = \lambda \langle H^0 \rangle^2.$$

The couplings of ϕ_R (ϕ_I) to F_1 and F_2 are respectively given by $Y_1/\sqrt{2}$ ($iY_1/\sqrt{2}$) and $Y_2/\sqrt{2}$ ($iY_2/\sqrt{2}$). Notice that $U(1)_{NEW}$ protects real and imaginary components of S as well as the neutral component of Φ' from such splitting. The λ' coupling leads to a mass term of form $\lambda' \langle H^0 \rangle^2 |\phi^-|^2$. Taking λ' positive, ϕ^- can be heavier than ϕ_I and ϕ_R so ϕ^- can decay to ϕ_R and/or ϕ_I .

Imposing both the Z_2 and $U(1)_{NEW}$ symmetries opens a possibility of having two DM candidates. The neutral components of F_i cannot be suitable dark matter candidates in this model because, as mentioned above, charged components of F_i^- are also degenerate with them and might lead to the presence of electrically charged DM. Thus, we take F_i heavy enough to decay to ψ and Φ . In this case, ϕ_I which is the lightest $U(1)_{NEW}$ neutral and Z_2 -odd particle will be stable and contribute to the dark matter abundance. If ϕ_I and ϕ_R are quasi-degenerate (*i.e.*, $(m_R - m_I)/m_R < 1/20$), their contribution to DM abundance will be suppressed within thermal freeze-out scenario. The electroweak singlet S can also kinematically be made stable and can therefore contribute to DM abundance. The annihilations of S will be then through the g_α and h_α couplings to $l\bar{l}$ pairs. The annihilation will be suppressed by $m_l^2/m_F^2 \ll 1$ where $m_F > \text{few } 100 \text{ GeV}$, so within this scenario, the density of S would overclose the universe. Thus, we take S heavy enough to decay into leptons and F_i .

We take the DM candidate to be the Dirac fermion, ψ . The Dirac field can annihilate to lepton and anti-lepton pair via $Y_{R\alpha}$ coupling with a cross section required within thermal freeze-out scenario. Notice that Φ' does not appear in the snail diagram. We have added this new scalar doublet to facilitate the annihilation of $\psi\bar{\psi}$ pair to lepton anti-lepton pairs via the $Y_{R\alpha}$ coupling. Instead of the $Y_{R\alpha}$ coupling, we could introduce a coupling of form $Y_{L\alpha} \Phi''^\dagger e_{R\alpha}^\dagger \psi_L$ where Φ'' is a $SU(2)$ singlet with electric charge equal to that of the electron. We have taken $Y_{R\alpha}$ coupling instead of $Y_{L\alpha}$ for definiteness. Replacing it with $Y_{L\alpha}$ does not change the discussion. Similarly, we could include new colored and charged scalar(s) to introduce Yukawa couplings to quarks and hence annihilation of dark matter pair to quarks. Studying all these possibilities and their potential signature at the LHC is beyond the scope of the present paper and will be done elsewhere. In summary, in our model DM is composed of ψ along with a subdominant contribution from ϕ_I .

The following remarks on the $U(1)_{NEW}$ symmetry are in order:

- The $U(1)_{NEW}$ not only protects the DM candidate from decay but it also protects the fermions (in particular ψ) from having Majorana mass. If ψ obtains even a tiny Majorana mass at loop level, it can be decomposed in terms of Majorana mass eigenstates $\psi_1 \equiv (\psi + \psi^c)/\sqrt{2}$ and $\psi_2 \equiv (\psi - \psi^c)/\sqrt{2}$ among which only the lighter one will survive and play the role of the dark matter. With Majorana dark matter, $\sigma(\psi_1\psi_1 \rightarrow l\bar{l})$ will be either p-wave suppressed or will be suppressed by $m_l^2/m_\Phi^2 \ll m_\psi^2/m_\Phi^2$, and cannot account for the observed DM abundance within the thermal freeze-out scenario.

- Notice that we have assigned opposite $U(1)_{NEW}$ charges to F_1 and F_2 that appear in the vertices connected to the external ν_α and ν_β lines. Without $U(1)_{NEW}$, we could drop F_2 and have a lepton number violating mass term of form $F_1^T c F_3$ giving a neutrino mass contribution proportional to $g_\alpha g_\beta$. This will not however help us to make the model more economic because a mass matrix proportional to $g_\alpha g_\beta$ has only one nonzero mass eigenvalue which cannot account for the realistic neutrino mass structure with at least two nonzero values. To reconstruct the neutrino mass matrix, another field with nonzero coupling component in the direction perpendicular to g_α in the flavor space is required.
- The $U(1)_{NEW}$ cannot be replaced with a Z_2 subgroup of it because Z_2 does not forbid Majorana mass for ψ . We could however invoke the Z_3 subgroup of $U(1)_{NEW}$ under which $\psi_L \rightarrow e^{\pm i2\pi/3} \psi_L$ and $\psi_R \rightarrow e^{\mp i2\pi/3} \psi_R$. For neutrino mass generation as well as DM consideration there is no significant difference between these two. The Z_3 symmetry allows terms such as S^3 but the $U(1)_{NEW}$ symmetry forbids them. The presence of such terms does not change our results. The reason why we have chosen $U(1)_{NEW}$ instead of Z_3 is that $U(1)_{NEW}$ can be eventually gauged to protect against symmetry breaking by quantum gravitational effects. Notice that only new particles are charged under $U(1)_{NEW}$. The gauged $U(1)_{NEW}$ can provide a way to have self-interacting DM, which provide a better fit to small scale features. A kinetic mixing of $U(1)_{NEW}$ with the photon can lead to a direct detection signal. We will not however try to gauge $U(1)_{NEW}$ here.

4 Lepton Flavor Violating rare decays

Before proceeding to discuss contribution to neutrino masses, dark matter abundance and effects at colliders, let us derive bounds on parameters from searches for LFV rare decays. The h_α and g_α couplings in Eq. (4) lead to Lepton Flavor Violating (LFV) rare decays, $l_\alpha \rightarrow l_\beta \gamma$ at one loop level. Using formulas in [15], we find that g_α coupling leads to

$$\Gamma(l_\alpha \rightarrow l_\beta \gamma) = g_\alpha^2 g_\beta^2 \frac{m_\alpha^5}{16\pi} \frac{[S(t)]^2}{(16\pi^2)^2 m_S^4} \quad (5)$$

where

$$S(t) = \frac{t-3}{4(t-1)^2} + \frac{\log t}{2(t-1)^3} + \frac{-2t^2+7t-11}{12(t-1)^3} + \frac{\log t}{2(t-1)^4} \quad (6)$$

in which $t \equiv (m_{F_1^-}/m_S)^2$. $S(t)$ is a monotonously decreasing function with $S(0) = 1/6$, $S(1) = 1/24$ and $S(\infty) = 1/12t$ so, as expected from decoupling theorem, $\Gamma(l_\alpha \rightarrow l_\beta \gamma)$ is suppressed by $1/(\text{Max}(m_S^2, m_{F_1^-}^2))^2$. The effect of the h_α coupling is given by the same formula replacing $g_\alpha, g_\beta \rightarrow h_\alpha, h_\beta$ and $m_{F_1^-} \rightarrow m_{F_2^-}$. If Φ' couples to more than one flavor, the $Y_{R\alpha}$ coupling can also lead to similar LFV effects. As mentioned before, to avoid LFV rare decays induced by $Y_{R\alpha}$, we assume Φ' couples only to one flavor. In the following, we discuss constraints on g_α from LFV bounds.

The best present bounds on LFV rare decay branching ratios are [16]

$$Br(\mu \rightarrow e \gamma) < 5.7 \times 10^{-13}, \quad (7)$$

$$Br(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8} \quad (8)$$

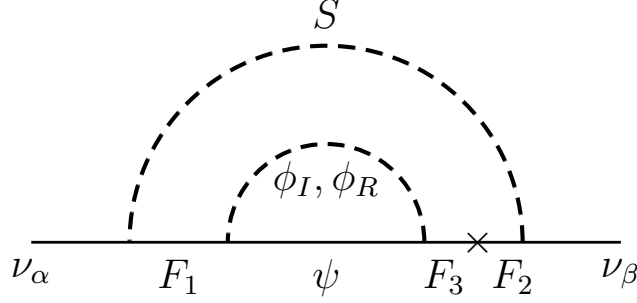


Figure 3: *Diagram giving mass to neutrinos. “×” indicates the m_M mass term insertion which violates lepton number conservation.*

and

$$Br(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8} . \quad (9)$$

From Eq. (7), we find

$$g_e g_\mu \lesssim 10^{-3} \frac{\text{Max}(m_S^2, m_{F_1^-}^2)}{\text{TeV}^2} \quad (10)$$

and from Eqs. (8,9), we find

$$g_e g_\tau, g_\mu g_\tau \lesssim \frac{\text{Max}(m_S^2, m_{F_1^-}^2)}{\text{TeV}^2} . \quad (11)$$

Similar consideration and bound hold valid for the h_α coupling, replacing $m_{F_1^-} \rightarrow m_{F_2^-}$.

5 Neutrino masses

For simplicity, let us set $Y'_1 = Y'_2 = 0$. Discussion for nonzero Y'_1 and Y'_2 will be similar. In this model, we have only one diagram contributing to neutrino mass. That is of form of snail diagram shown in Fig (2-d), where S_1 , S_2 and F_4 should be respectively identified with S , Φ^0 and ψ of our model. Instead of using $\lambda \langle H \rangle^2 (\Phi^0)^2$ mass insertion approximation, we can have mass eigenstates ϕ_I and ϕ_R (imaginary and real components of Φ^0) propagating in the inner loop as shown in Fig. 3. Going to mass basis ϕ_R and ϕ_I , the contribution of these fields propagating in the inner loop will be respectively given by factors $(Y_1/\sqrt{2})(Y_2/\sqrt{2})[1/(p^2 - m_R^2)]$ and $(iY_1/\sqrt{2})(iY_2/\sqrt{2})[1/(p^2 - m_I^2)]$ so the sum of two contributions will be proportional to

$$\frac{Y_1 Y_2 (m_R^2 - m_I^2)}{2(p^2 - m_I^2)(p^2 - m_R^2)} .$$

We use mass insertion approximation for $\langle F_3 F_2^T \rangle$ propagator: $k^2 m_M / [(k^2 - m_{F_3}^2)(k^2 - m_{F_2}^2)]$. Putting all these together we find that the two-loop snail diagram contribution to neutrino mass is given by

$$(m_\nu)_{\alpha\beta} = (g_\alpha h_\beta + g_\beta h_\alpha) m_M \frac{Y_1 Y_2}{2} (m_R^2 - m_I^2) \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 k}{(2\pi)^4}$$

$$\frac{1}{k^2 - m_S^2} \frac{k \cdot \sigma}{k^2 - m_{F_1}^2} \frac{(p+k) \cdot \bar{\sigma}}{(k+p)^2 - m_\psi^2} \frac{1}{(p^2 - m_R^2)(p^2 - m_I^2)} \frac{k^2}{(k^2 - m_{F_2}^2)(k^2 - m_{F_3}^2)}.$$

Without loss of generality, we can go to a basis where g_α takes the form of $(0, 0, g)$. We still have the freedom to rotate h_α in the direction $(0, h_1, h_2)$. In this basis, the first row and column of m_ν vanishes so with this field content one of neutrino mass eigenvalues will be zero. The mass scheme will be therefore hierarchical but the mixing parameters and CP-phases can be reconstructed with proper choice of g_α and h_β . To obtain non-hierarchical scheme, we can add another singlet S coupled to L . Using Feynman parameters we find

$$(m_\nu)_{\alpha\beta} = \frac{(g_\alpha h_\beta + g_\beta h_\alpha)}{16} m_M Y_1 Y_2 \frac{(m_R^2 - m_I^2)}{(16\pi^2)^2} I(m_{F_1}, m_{F_2}, m_{F_3}, m_S, m_\psi, m_I, m_R)$$

where $I(m_{F_1}, m_{F_2}, m_{F_3}, m_S, m_\psi, m_I, m_R)$ is defined as

$$\int_0^1 dy \int_0^{1-y} dx \int_0^1 da_1 \int_0^{1-a_1} da_2 \int_0^{1-a_1-a_2} da_3 \int_0^{1-a_1-a_2-a_3} da_4 \frac{1-x}{A}$$

in which A is equal to

$$(a_1 m_{F_1}^2 + a_2 m_{F_2}^2 + a_3 m_{F_3}^2 + a_4 m_S^2)x(1-x) + (1-a_1-a_2-a_3-a_4)(x m_\psi^2 + y m_I^2 + (1-x-y)m_R^2).$$

Notice that A is a positive definitive quantity over the whole integration range. Thus, the integration I is a finite quantity as expected. ψ is the lightest field propagating in the loops. Let us denote the mass of the heaviest field propagating in the loop by m_{max} . We can then write $I(m_{F_1}, m_{F_2}, m_{F_3}, m_S, m_\psi, m_I, m_R) = b/m_{max}^2$ where b is a number. For m_ψ/m_{max} (and therefore the rest of ratios) varying between ~ 0.1 to 1, the value of b varies in the range $O(0.01)$ - $O(0.1)$. The neutrino mass can be then estimated as

$$m_\nu \sim (0.01 - 0.1 \text{ eV}) Y_1 Y_2 \frac{g \times h}{10^{-1} \times 10^{-2}} \frac{m_M}{5 \text{ GeV}} \frac{(m_R^2 - m_I^2)/m_{max}^2}{1/20}. \quad (12)$$

Notice that $m_R^2 - m_I^2 \sim \lambda \langle H^0 \rangle^2$. Taking $\lambda \sim 0.5$ and $m_{NEW} \sim (\text{few TeV})$, it seems to be natural to have $(m_R^2 - m_I^2)/m_R^2 \leq (m_R^2 - m_I^2)/m_{NEW}^2 \lesssim 0.1$. As we will discuss in sec. 6, $(m_R - m_I)/m_R$ should be smaller than ~ 0.05 to facilitate the coannihilation of ϕ_I and ϕ_R (*e.g.*, $\phi_I \phi_R \rightarrow Z^* \rightarrow SM$) in the early universe and hence prevent over-closure of the universe by lighter component of ϕ_I and ϕ_R .

The following points are in order:

- To make the estimate in Eq. (12), we have taken $g_\alpha h_\beta \sim 10^{-3}$. As we saw see in section 4, for $m_{NEW} \sim 1 \text{ TeV}$, the upper bounds on $g_e g_\mu$ and $h_e h_\mu$ from $\text{Br}(\mu \rightarrow e\gamma)$ are of order of 10^{-3} so we expect an observable effect in near future at searches for $\mu \rightarrow e\gamma$. Within this model, saturating bounds on $\text{Br}(\tau \rightarrow \mu\gamma)$ or $\text{Br}(\tau \rightarrow e\gamma)$ can be possible only if $g_\alpha \sim 10^{-3} \ll h_\alpha \sim 1$ or $g_\alpha \sim 10^{-3} \gg h_\alpha \sim 1$.
- To arrive at Eq. (12), we have used mass insertion approximation for the treatment of mass term mixing F_2 and F_3 , m_M . Taking $m_M = 5 \text{ GeV}$ and $m_{F_i} \sim \text{TeV}$, this approximation is valid. Taking smaller m_M requires $Y_1, Y_2 \gg 1$ which leads to non-perturbativity.

- In the range $m_\Phi \sim m_F \sim m_{max} \sim 1 \text{ TeV} - 100 \text{ TeV}$ and $m_M \sim 5 \text{ GeV} (m_{max}^2/\text{TeV}^2)$, we obtain desired values of m_ν satisfying bounds from LFV as well as collider searches and we still remain in the perturbativity range: $Y_1, Y_2 < 1$ and $m_M \ll m_F$. The lower part of this range can be probed at second phase of the LHC, but the range $m_F, m_\phi > 10 \text{ TeV}$ is out of the reach of the LHC.

6 Annihilation to lepton pair

As discussed in section 3, we choose the main dark matter component to be ψ which annihilates to a pair of leptons. The annihilation cross section to a charged lepton pair of flavor α can be written as

$$\langle \sigma(\psi\bar{\psi} \rightarrow \ell_\alpha \bar{\ell}_\alpha) v \rangle = \frac{|Y_{R\alpha}|^4}{32\pi} \frac{m_\psi^2}{(m_\psi^2 + (m_{\phi'^-})^2)}. \quad (13)$$

A similar equation can be rewritten for annihilation to a $\nu_\alpha \bar{\nu}_\alpha$ pair by replacing $m_{\phi'^-}$ with $m_{\phi'^0}$. To avoid large LFV effects, we assume that only one flavor component of $Y_{R\alpha}$ is nonzero. Taking $\langle \sigma_{tot} v \rangle = 3 \times 10^{-26} \text{ cm}^3 \text{sec}^{-1}$ (as predicted within the thermal freeze-out scenario) and typical values $m_\psi = 300 \text{ GeV}$ and $m_{\phi'^-} = m_{\phi'^0} = 400 \text{ GeV}$ we find $Y_R = 0.55$. In general, we obtain

$$m_{\phi'^-}, m_{\phi'^0} \leq 1.4 Y_{R\alpha}^2 \text{ TeV} \quad (14)$$

where equality corresponds to the limiting case of $m_\psi \rightarrow m_{\phi'^-} \simeq m_{\phi'^0}$.

The large $Y_{R\alpha}$ coupling will not however affect the lepton or heavy meson decays because they are not heavy enough to emit ψ . This large coupling can cause dips in the spectrum of very high energy cosmic neutrinos at ICECUBE due to scattering off the DM distributed all over the universe. The resonance energy is at $E_{res} \sim (m_{\phi'^0})^2/m_\psi \sim \text{few } 100 \text{ GeV}$. For a given $m_{\phi'}$, decreasing m_ψ , the value of E_{res} and as a result the position of the dip shifts towards higher energies. One should however bear in mind that by decreasing $m_\psi/m_{\phi'}$ the required Y_R increases and eventually enters non-perturbative regime.

Data from the region close to galaxy center from Fermi-LAT shows a hint of GeV range gamma excess. One of the explanations is the annihilation of 10 GeV DM pairs to lepton pairs [9]. It is tantalizing to try to accommodate this signal within our model. Now, following Ref [17], if we set $\langle \sigma(\psi\bar{\psi} \rightarrow l\bar{l}) v \rangle = 0.86 \times 10^{-26} \text{ cm}^3 \text{sec}^{-1}$ and $m_\psi \sim 10 \text{ GeV}$, we obtain

$$Y_R = 0.5(m_{\phi'^-}/100 \text{ GeV})(10 \text{ GeV}/m_\psi)^{1/2}.$$

Notice that we have taken ϕ' to be relatively light. From the first run of the LHC there, there is already a lower bound of 325 GeV on the mass of new charged scalar such as ϕ'^- whose decay lead to the electron or the muon plus missing energy [18]. Bounds on such scalar coupled to only tau is weaker: $m_{\phi'^-} > 90 \text{ GeV}$ [19]. As a result, for annihilation to tau pair, the value of $m_{\phi'^-}$ satisfies the present bound. For heavier values of ϕ'^- , we eventually enter non-perturbative regime. A more recent analysis of the gamma ray excess finds a better fit with $m_\psi \sim 50 \text{ GeV}$ and $\langle \sigma(\psi\bar{\psi} \rightarrow b\bar{b}) \rangle \sim 10^{-26} \text{ cm}^3 \text{sec}^{-1}$ [10]. This can be achieved with a coupling of form $Y_b \bar{b}_R \psi \phi''$ where ϕ'' is a colored and charged scalar singlet under $SU(2)$. From the LHC bounds, this scalar should be heavier than 620 GeV [20]. The annihilation cross section of $\psi\bar{\psi} \rightarrow b\bar{b}$ is given by Eq. (13) replacing ϕ' with ϕ'' and multiplying by a factor of three to account for the

color degrees of freedom. To accommodate the signal with m_ψ and $m_{\phi''} \sim 700$ GeV, Y_b should be of order of one. One should however bear in mind that DM origin of gamma ray excess has been questioned in a series of publication [11].

As discussed before the lightest neutral component of ϕ (*i.e.*, ϕ_I) can be another DM component if it is lighter than ϕ^- . For $|m_R - m_I| \lesssim m_R/20$, coannihilation via $\phi_I \phi_R \rightarrow Z^* \rightarrow SM$ will render its abundance negligible.

7 Signature at the LHC

In this model, there are several fields with electroweak interactions that can be pair produced at the LHC provided that they are light enough. As discussed in sec. 5, Φ and F_i fields propagating in the loops that contribute to m_ν can have masses in the range 1 TeV-100 TeV. As discussed in sect. 3, we take Φ to be lighter than F_1 and F_3 . As result, via large Y_1 and Y_2 couplings, the components of F_2 and F_3 will decay as $F_i^- \rightarrow \psi \phi^-$ and $F_i^0 \rightarrow \psi \phi_{I(R)}^0$. The ψ particle as well as ϕ_I will appear as missing energy. Via tree-level Z^* exchange, $\phi_R \rightarrow \phi_I \nu \bar{\nu}$, $\phi_I l \bar{l}$.

While Φ and F_i particles can be too heavy to be produced at the LHC, as we saw in sect. 6, there is an upper bound on the masses of the Φ' components. Thus, if this model is realized in nature, it is guaranteed that the components of Φ' will be pair produced at the second run of the LHC, leading to the following signals:

- Mono-lepton plus missing energy signal through $u\bar{d} \rightarrow \phi'^+ \phi'^0 \rightarrow (l^+ \psi)(\nu \bar{\psi})$ and the charge conjugate processes.
- Two-lepton plus missing energy signal through $u\bar{u}, d\bar{d} \rightarrow \phi'^+ \phi'^- \rightarrow (l^+ \psi)(l^- \bar{\psi})$.
- Missing energy through $u\bar{u}, d\bar{d} \rightarrow \phi'^0 \bar{\phi}'^0 \rightarrow (\bar{\nu} \psi)(\nu \bar{\psi})$.

As discussed in section 6, the present lower bounds on the masses of scalars whose decay lead to missing energy plus muon and electron [18] and tau lepton [19] are respectively 325 GeV and 90 GeV. In fact, phenomenology of Φ' doublet at the LHC (both production mechanism as well as signature of the decay product) is very similar to that of left-handed slepton doublet in the framework of Minimal Supersymmetric Standard Model (MSSM). As mentioned before, we assume ϕ' to couple mainly to only one flavor to avoid LFV rare processes. If this flavor happens to be the second generation, the signals at the LHC will be cleaner. In this case, we expect a contribution to $(g-2)_\mu$ which we elaborate on in the next section.

8 Muon magnetic dipole moment

In this model, there are several particles that couple to the muon and can give rise to $(g-2)_\mu$ at one loop level. Considering the bounds in Eq. (14) on the mass and coupling of ϕ' , it can give largest contribution to $(g-2)_\mu$ if the $Y_{R\alpha}$ coupling is to the muon flavor. The $Y_{R\mu}$ coupling leads to

$$\delta \frac{g-2}{2} = \frac{Y_{R\mu}^2}{16\pi^2} \frac{m_\mu^2}{m_{\phi'^-}^2} K(r)$$

where

$$K(r) = \frac{2r^2 + 5r - 1}{12(r-1)^3} - \frac{r^2 \log r}{2(r-1)^4}$$

in which $r = (m_\psi^2/m_{\phi'}^2)$. Taking $m_{\phi'} \sim 100 \text{ GeV} - 1 \text{ TeV}$ and $Y_{R\alpha} \sim 1$ (see Eq. 14), we find that $(g-2)_\mu/2 \sim 10^{-11} - 10^{-12}$ which is well below the current sensitivity limit [16].

9 Conclusions

Following the “recipes” developed in [2], we have built a model in which neutrinos receive Majorana mass via a two-loop diagram with topology of “snail diagram” depicted in Fig. 2-d and in Fig. 3. The particles propagating in the loops are new scalars and fermions charged under $SU(2) \times U(1)$. The field content is given in table 1. The lepton number is explicitly broken by fermion mass m_M (see Eq. 3) so the neutrino masses are proportional to m_M as seen in Eq. (12). Following the argument in Ref. [2], we confirm that the two-loop snail diagram is the leading contribution to neutrino mass. Within this model the neutrino mass scheme is predicted to be hierarchical with one vanishing mass eigenvalue. The model respects a global $U(1)_{NEW} \times Z_2$ symmetry which stabilizes two of new particles: ϕ_I , the imaginary part of the neutral component of Φ and ψ , a singlet under electroweak group. We assume the mass splitting between ϕ_I and ϕ_R (the real component of ϕ^0) is small enough to allow efficient co-annihilation in the early universe. ϕ_I is therefore only a sub-dominant component of dark matter. This assumption turns out to be natural within our model and does not need any fine-tuning.

The dominant component of dark matter is Dirac fermions ψ that can annihilate to a pair of standard model fermions via a Yukawa coupling involving new scalar Φ' . In order to obtain the observed abundance of dark matter within freeze-out scenario (*i.e.*, $\langle \sigma(\psi\bar{\psi} \rightarrow f\bar{f})v \rangle \sim 1 \text{ pb}$), the mass of Φ' should be less than 1.5 TeV (see Eq. (14)). This means the components of Φ' can be eventually produced at the LHC via electroweak interactions and discovered through their signature of decay to standard model fermions plus missing energy. Moreover the corresponding Yukawa coupling should be of order of one. To avoid LFV rare decay, we assume Φ' couples only to one flavor. If this flavor is the muon, the discovery potential of the LHC will be higher. The contribution to $(g-2)_\mu$ is then predicted to be one or two orders of magnitude below the present sensitivity. The coupling of the scalar singlet, S to leptons (*i.e.*, g_α and h_α) should involve more than one flavor to reconstruct the neutrino mass matrix structure. This in turn leads to LFV rare decays. From values of neutrino mass, we expect the $\mu \rightarrow e\gamma$ signal to be around the corner.

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